«Smoothed Particle Hydrodynamics» Applications to Astrophysical Simulations

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SciNet User Group (SNUG) Meeting

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Outline

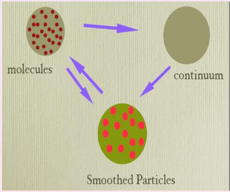
1. Smoothed Particle Hydrodynamics
   - Basic Ideas
   - Application to Hydrodynamics
   - Examples & Applications
2. Astrophysical Applications
   - Code Details
   - Accretion Disk+Recoiled BH
   - *V838 Monocerotis* Outburst
3. Conclusions
   - Discussion
   - References

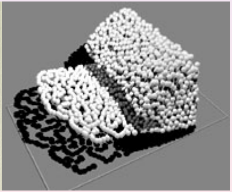
SPH: Basic Concepts

replace the continuum by discrete (moving) elements/points/particles

- Smoothed-Particle Hydrodynamics (SPH)
  - is a computational method used for simulating fluid flows.
  - has been used in astrophysics, ballistics, vulcanology, oceanology, ...
  - is a mesh-free Lagrangian method (where the coordinates move with the fluid), and the resolution of the method can easily be adjusted with respect to variables such as the density.

- (SPH) method 'divides' the fluid into a set of discrete elements: «particles»
  - ➠ particles ⇝ spatial distance: "smoothing length" h / their props. are "smoothed" by a «kernel function».





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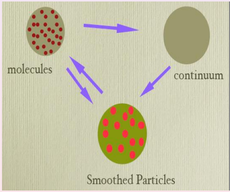

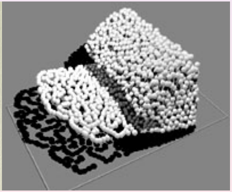

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Smoothing kernel function and General properties

- integral quantities ➡ replace delta-fn by «kernel fn» (kernel approx.)

  $f(\vec{x}) = \int_\Omega g(\vec{x'})\delta(\vec{x}-\vec{x'})d\vec{x'}$ ➡ $f(\vec{x}) = \int_\Omega g(\vec{x'})W(\vec{x}-\vec{x'},h)d\vec{x'}$

  ➡ mathematical props: normalized, compact support (smoothing length h), differentiable.

- density: $\rho_i(\vec{r}) = m_i W(|\vec{r}-\vec{r}_i|,h)$ $\rho(\vec{r}) = \sum_j m_j W(|\vec{r}-\vec{r}_j|,h)$

- hydrodynamics quantities q, $q_i = \sum_j \frac{q_j}{\rho_j} W(|\vec{r}-\vec{r}_j|,h)$
  - any extensive quantity can be calculated as a density weighted sum.

  ➡ derivatives in SPH: $\nabla q_i = \sum_j m_j \frac{q_j}{\rho_j} \nabla W(|\vec{r}-\vec{r}_j|,h)$

- fundamental eqns: $\langle \nabla \cdot f(\vec{c}) \rangle = -\int_\Omega f(\vec{x'}) \cdot \nabla W(|\vec{r}-\vec{r}_j|,h)$

  ➡ $m_j = \Delta V_j \rho_j$ ➡ $\langle f(\vec{x}_i) \rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} \cdot W(|\vec{x}_i - \vec{x}_j|,h)$

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Navier-Stokes equation

The SPH formulation is derived by spatially discretizing the Navier-Stokes equations

➨ dissipative viscous forces (similar to friction), changes in pressure, gravity, and other forces acting inside the fluid

General SPH formalism for dynamical flow also includes

- Artificial viscosity
- Artificial heat

- Continuity eqn.:
  $\frac{d\rho_i}{dt} = \sum_j m_j(\vec{v_i} - \vec{v_j}) \cdot \nabla_i W(\vec{r_i} - \vec{r_j}, h)$
- Non-dissipative

$$\frac{d\vec{v_i}}{dt} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W_{ij}(h) + \vec{g_i}$$

- Viscous fluids

$$\Pi_{ij} = -\frac{\alpha \bar{c}_{ij}}{\bar{\rho}_{ij}} \frac{\vec{v}_{ij} \cdot \vec{r}_{ij}}{|\vec{r}_{ij}|} \rightsquigarrow -\frac{16\nu_i\nu_j}{(\nu_i\rho_i + \nu_j\rho_j)} \frac{\vec{v}_{ij} \cdot \vec{r}_{ij}}{\bar{h}_{ij}|\vec{r}_{ij}|}$$

- Boundaries

$$+\sum_k \left[\vec{f}_{ik} - m_k \Pi_{ik} \nabla_i W_{ik}(h) \right]$$

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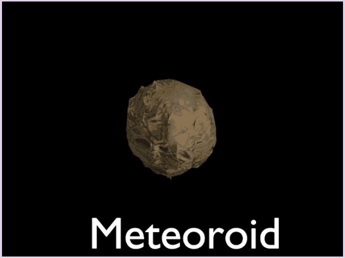
Astrophysics: Galaxies

➧ SRC: http://nylander.wordpress.com/category/physics/

Fluids

(astro)-geology



SRC: http://en.wikipedia.org/wiki/Meteor#Meteor

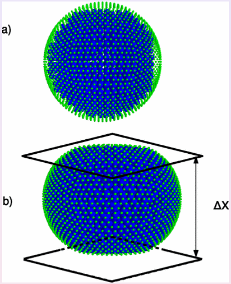
games – NVIDIA package PhysX

SRC: https://developer.nvidia.com/sites/akamai/physx/Index.html
physXinfo.com

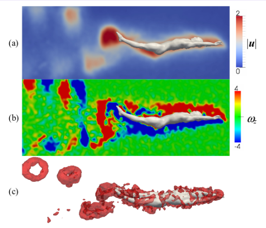
SRC: DualSPHhysics – dual.sphysics.org § www.youtube.com/user/DualSPHysics

Other examples and applications
[Annu.Rev.Fluid Mech. 44 (2012), J.J.Monaghan]

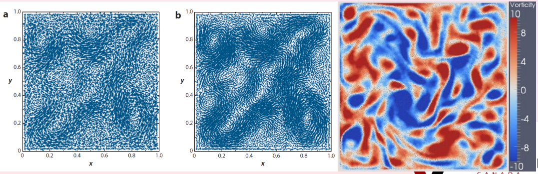
- Dam Breaks & Plunging Waves
- Gravity Currents, Multifluid Phenomena
- Bodies Moving in Fluids
- Non-Newtonian Fluids
- Surface Tension
- Diffusion and Precipitation
- Elasticity, Fracture, Soft Tissue
- Dust, Granular Flow, and Granicles
- Turbulence



Van Liedekerke *et al.* PRE 81 (2010)



Cohen, Clearly, Mason, IFBME Proc.31 (2010)



M.Robinson and J.J.Monaghan (2011)

"A Parallel Smoothed Particle Hydrodynamics Code for Calculating Stellar Interactions" – [J.Faber, J.Lombardi, F.Rasio.]: «`StarCrash`»

- parallelized code (serial version).
- kernel fn. $W(r,h) = \frac{1}{\pi h^3} \begin{cases} 1 - \frac{3}{2}\left(\frac{r}{h}\right)^2 + \frac{3}{4}\left(\frac{r}{h}\right)^3, & 0 \leqslant \frac{r}{h} < 1, \\ \frac{1}{4}\left[2 - \frac{r}{h}\right]^3, & 1 \leqslant \frac{r}{h} < 2, \\ 0, & \frac{r}{h} \geqslant 2. \end{cases}$

- Originally developed for study of stars interactions (i.e. binary neutron stars).

- Evolution in time:
  - time step satisfies Courant stability condition (sound crossing time for a particle $-h_i/c_S-$ & accelerative timescale for a particle $-\sqrt{(h_i/a_i)}-$).
  - second-order "leapfrog" approach.

- Gravity solver
  - gravitational potential,

$$\Phi(\vec{r}) = \int \rho(\vec{r'}) \frac{d^3\vec{r'}}{|\vec{r}-\vec{r'}|} = \sum_i \frac{m_i}{|\vec{r}-\vec{r_i}|}$$

  - gravitational force in each particle (direct summation $\sim N^2$)
  - FFT convolution (`FFTW` library)

$$\Phi(\vec{r}) = \text{FFT}^{-1} \times \left[\text{FFT}(\rho) * \text{FFT}\left(\frac{1}{r}\right)\right]$$

  - 3d gravity grid.

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Code Details

Variable smoothing length & “extreme”-physics

- “grad-h” Lagrangian derived formulation conserves energy to tenths of a percent over full history

[Monaghan & Price (2001); Springel & Hernquist (2002)]

➠ «grad-h» correction factor,

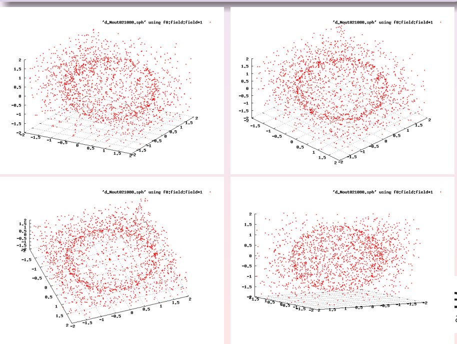
$$\tilde{\Omega}_b \equiv 1 - \frac{\partial h_b}{\partial N_b} \sum_k \frac{\partial W_{bk}(h_b)}{\partial h_b}$$

$$N_b = \sum_k \nu_k W\left(|\vec{r_b} - \vec{r_k}|, h_b\right)$$

- Shock heating was included
- Gravity calculations were done using a N-body library on GPUs (cuN-body)

- BH softening potential at smoothinglength scales

W/o softening, mutual repulsion of particles near BH leads to quasistable but unphysical “bubbles”



Code (technical) Details

Evolution Equations

- Lagrangian [Monaghan & Price (2007)]

$$L_{\rm grav} = -\sum_i m_i \Phi_i = -GM_{\rm BH} \sum_i m_i \varphi_i(h_i)$$

$$\varphi(r,h) = \begin{cases} \left(-\frac{7}{5} + \frac{2}{3}q^2 - \frac{3}{10}q^4 + \frac{1}{10}q^5\right)/h, & 0 \leqslant q < 1; \\ \left(-\frac{8}{5} + \frac{1}{15q} + \frac{4}{3}q^2 - q^3 + \frac{3}{10}q^4 - \frac{1}{30}q^5\right)/h, & 1 \leqslant q < 2; \quad q \equiv \frac{r}{h}, \\ -1/r & q \geqslant 2 \end{cases}$$

$$\dot{\mathbf{v}}_i^{(\rm grav)} = -GM_{\rm BH}\nabla_i\varphi_i(h_i) - GM_{\rm BH}\sum_j m_j \left[\frac{1}{\Omega_i}\frac{\partial\varphi_i}{\partial h_i}\frac{\partial h_i}{\partial\rho_i}\nabla_i W_{ij}(h_i) + \frac{1}{\Omega_j}\frac{\partial\varphi_j}{\partial h_j}\frac{\partial h_j}{\partial\rho_j}\nabla_i W_{ij}(h_j)\right]$$

Code (technical) Details

Thermodynamics

monoatomic ideal gas

$$E_{\rm INT} = \sum_i \frac{1}{\gamma-1} m_i A_i \rho_i^{\gamma-1} = \frac{3}{2}\sum_i m_i A_i \rho_i^{2/3}$$

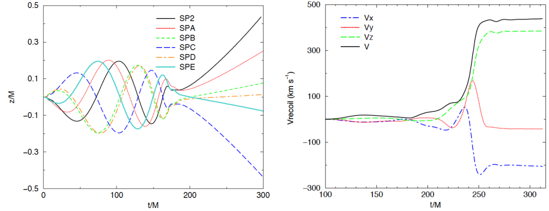
the temperature is related to the specific internal energy $u_i = \frac{3}{2} A_i \rho_i^{2/3}$ via the ideal gas law $E_{\rm INT} = \frac{3}{2} k_B T$

$$T_i = \frac{2}{3k_B}\mu m_p u_i = \frac{\mu m_p}{k_B} A_i \rho_i^{2/3} = \frac{\mu m_p}{k_B}\frac{P_i}{\rho_i},$$

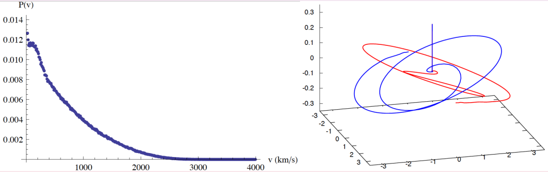
where m_p is the mass of a proton and $\mu = 0.617$ is the mean molecular weight, assuming that the disk is a plasma with mass fractions $X = 0.7,\ Y = 0.28$.

Binary SMBH mergers, simulations

- Beginning in 2005, the first successful BH binary mergers were calculated [Pretorious, Campanelli, ...]
- The inclusion of spins yielded potential kicks of 4000 km/s, much larger than the maximum from mass asymmetry alone [Campanelli et al. 2007]
- Accretion torques aligning the spin/orbit [Bogdanovic et al. 2007] typically yield smaller kicks ("wet" mergers)
- Lousto et al. (2010): Avg kick of 630 km/s for uniform mass ratio distribution and isotropic spins



Kick dependence on initial spin alignment [Campanelli et al. (2007)]



[Lousto et al. (2010)]

$$\vec{V}_{\rm recoil}(q,\vec{\alpha}) = v_m\,\hat{e}_1 + v_\perp(\cos\xi\,\hat{e}_1 + \sin\xi\,\hat{e}_2) + v_\|\,\hat{n}_\|,$$

$$v_m = A\frac{\eta^2(1-q)}{(1+q)}\left[1+B\,\eta\right],$$

$$v_\perp = H\frac{\eta^2}{(1+q)}\left[(1+B_H\,\eta)\,(\alpha_2^\| - q\alpha_1^\|) + H_S\,\frac{(1-q)}{(1+q)^2}\,(\alpha_2^\| + q^2\alpha_1^\|)\right],$$

$$v_\| = K\frac{\eta^2}{(1+q)}\Big[(1+B_K\,\eta)\,\left|\alpha_2^\perp - q\alpha_1^\perp\right|\cos(\Theta_\Delta-\Theta_0) + K_S\,\frac{(1-q)}{(1+q)^2}\,\left|\alpha_2^\perp + q^2\alpha_1^\perp\right|\cos(\Theta_S-\Theta_1)\Big],$$

Binary SMBH mergers, observational evidence

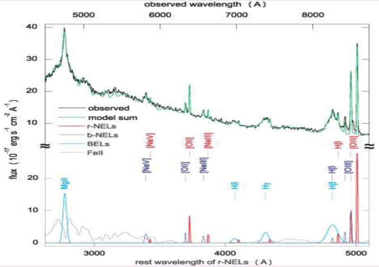
- Komossa et al. (2008): 2650 km/s blueshift of broad lines w.r.t. narrow lines in the AGN SDSSJ092712+294344.0
- Many Models proposed:
  - Binary merger recoil
  - AGN from small SMBH in binary

    [Bogdanovic et al. (2009); Dotti et al. (2009)]
  - SMBHs in pair of interacting galaxies

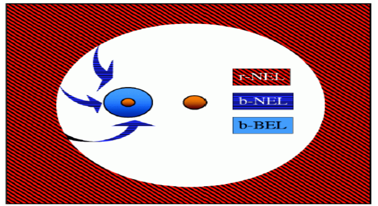
    [Heckman et al. (2009)]
  - Chance Superposition [Shields et al. (2009)]
- Other sources w/large velocity differences:
  - J105041.35+345631.3 - 3500 km/s,

    [Shields et al. (2009)]
  - E1821+643 - 2100km/s, [Robinson et al. (2010)]
  - CXOCJ100043.1+020637 - 1200km/s,

    [Civano et al. (2010)]



SDSSJ092712+294344.0 [Komossa et al. (2008)]



[Bogdanovic et al. (2009)]

Initial Data & Parameters

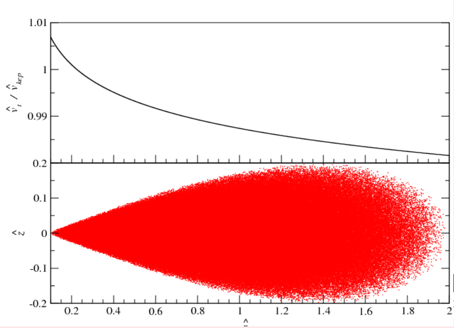
- Initial model: relaxed, nearly Keplerian, pressure-supported, non-self gravitating disk
- Inner radius at $\hat{r} = 0.1$, assuming decoupling of SMBHs from inner disk at late times [Schnittman & Krolik (2008)]
- Density maximum occurs near inner edge of disk
- Outer radius at $\hat{r} = 2$, maximum disk height $\hat{z} = 0.2$, nearly constant opening angle
- $N = 500,000$ particles per run
- Relaxed for long times, then brief dynamical phase; followed by either no kick or kicks of 15, 30, 45, 60 degrees from vertical

- integrating the force equation for a system in stationary equilibrium,

$$-\frac{\nabla P}{\rho} + \nabla\left(\frac{GM}{r}\right) = -\frac{l(r_c)^2}{r_c^3}\hat{\mathbf{r}}_\mathbf{c},\ P = k\rho^\gamma$$

Keplerian profile $l(r_c) = \sqrt{GMr_c}$,

$$z(r_c) = \sqrt{\left(\frac{GM}{cr_c^\alpha + K}\right)^2 - r_c^2}$$

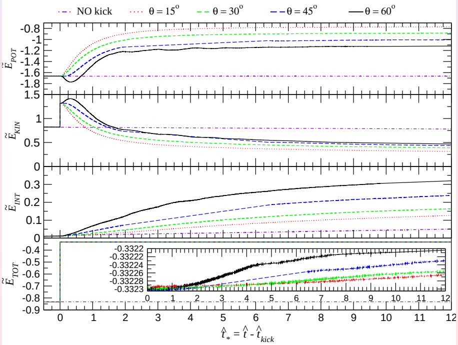


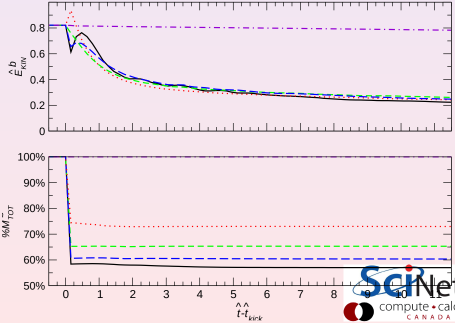
Energies

- Energies very quickly show a clear pattern; more oblique kicks
  - Yield smaller masses with our initial density profile
  - Have much higher internal energies

- Binding criterion

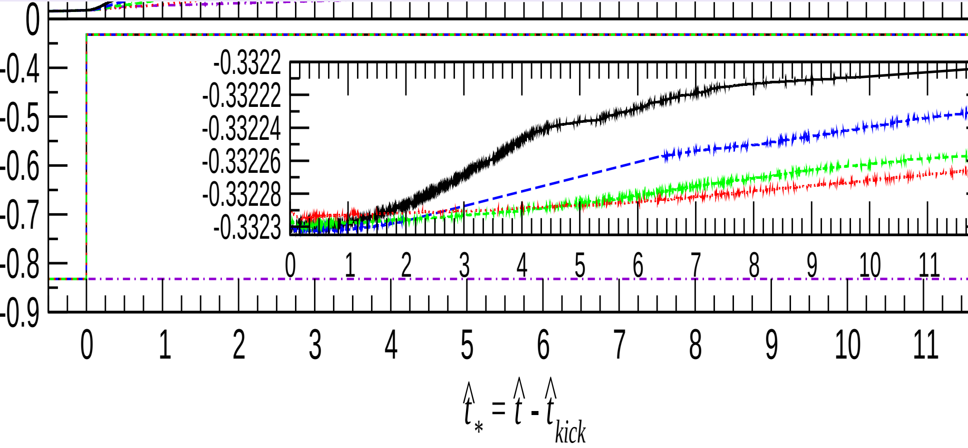
$$E_{\mathrm{POT},i} + E_{\mathrm{KIN},i} + E_{\mathrm{INT},i} = m_i \left(\Phi_i + \frac{|v_i|^2}{2} + \frac{3A_i\rho_i^{2/3}}{2} \right) < 0,$$





Energy Conservation

[MP, Faber, Lombardi – ApJ. 745 (2012) 71]



Movies available at http://lanl.arxiv.org/abs/1107.1711

$\theta = 15^\circ$

$\theta = 60^\circ$

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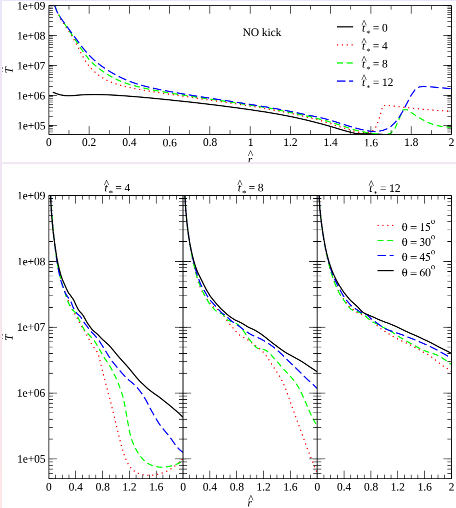
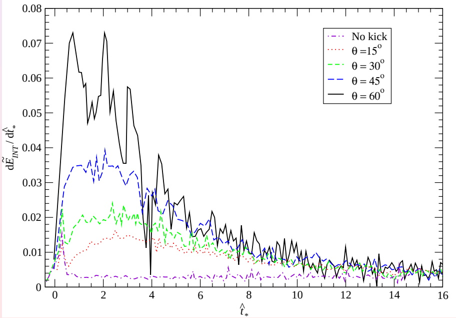
Temperatures & Luminosity



- Temperature distrib. nearly universal
- Internal energy differences reflect variation in density profile, not temperature profile
- Oblique kicks yield modulated luminosity in time (time derivative of internal energy)



Conclusions & Discussion || Summary

- Kick angle affects bound disk mass, luminosity in time, disk tilt, etc.
- The more oblique the kick, the more rapidly we fill the "gap" at the inner edge of the accretion disk
- Tremendous reserve of energy from accreting material, but we need much better resolution to study it accurately, as does anyone...
- SPH code optimized for accurate long-term simulations of disks around BHs

TABLE 1
SUMMARY OF THE RUNS PERFORMED.

Kick angle (°)	Bound mass	$\lvert\vec{L}_b\rvert$	$\vec{L}_b$	Tilt angle (°)	Max. Luminosity
15	0.73	0.543	(0.135,0.009,0.526)	14.3	0.016
30	0.65	0.444	(0.166, 0.011, 0.412)	21.9	0.024
45	0.60	0.359	(0.146, 0.010, 0.327)	24.2	0.039
60	0.57	0.296	(0.107, 0.006, 0.276)	21.3	0.073
None	0.9999	0.830	a	a	0.015/0.004[b]

▲ [MP, Faber, Lombardi – ApJ. 745 (2012) 71]

Ref. model: $m_{disk} = 10^4 M_\odot$, $M_{BH} = 10^8 M_\odot$ & $v_{kick} = 1000$km/s

- charact. timescales $\sim$ 1000 years
- $\mathcal{L} \sim 10^{42}$ erg/s
- $T \sim 10^{8-9}$ K
- should work for a wide range of masses and kicks veloc.

$v_{kick} \rightsquigarrow \frac{1}{2} v_{kick}$

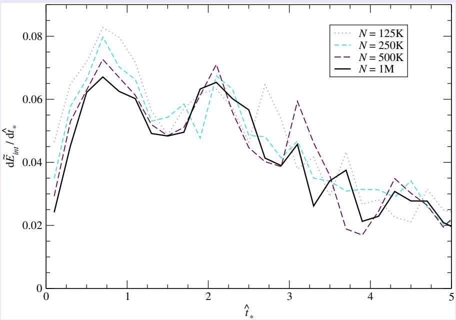
➠ $t \rightsquigarrow t \times 8$; $E, T \rightsquigarrow \times \frac{1}{4}$; $\mathcal{L} \rightsquigarrow \times \frac{1}{32}$

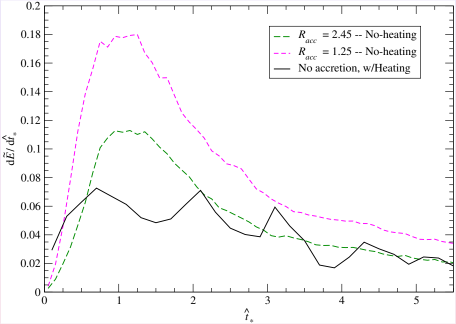
➠ $\therefore$ observational bias toward large kicks

TABLE 2
ESTIMATED ENERGIES AVAILABLE TO THE DISK

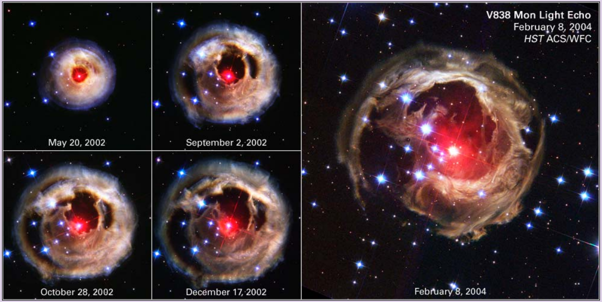
Kick angle (°)	Circularization energy	Internal energy at $\hat{t}_* = 12$	Internal energy, corrected for kick
15	0.10	0.12	0.07
30	0.09	0.16	0.11
45	0.16	0.24	0.19
60	0.43	0.32	0.27

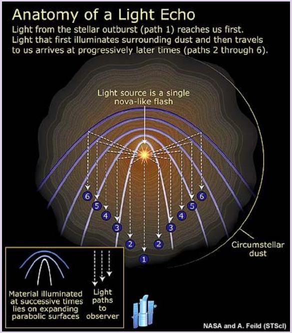
Convergence & Consistency Analysis





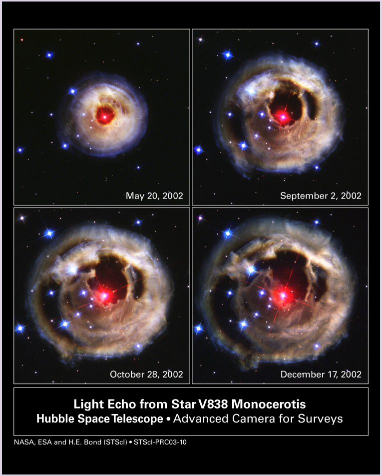
- **[MP, Faber, Lombardi – ApJ. 745 (2012) 71]**
- **[Rossi, Lodato, *et al.* – MNRAS 401 (2010) 2021]**



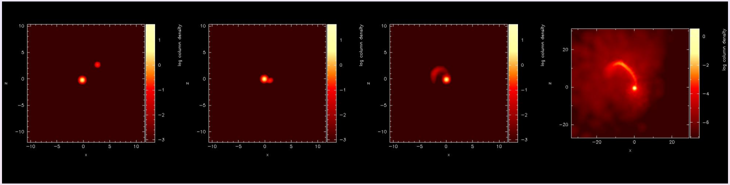


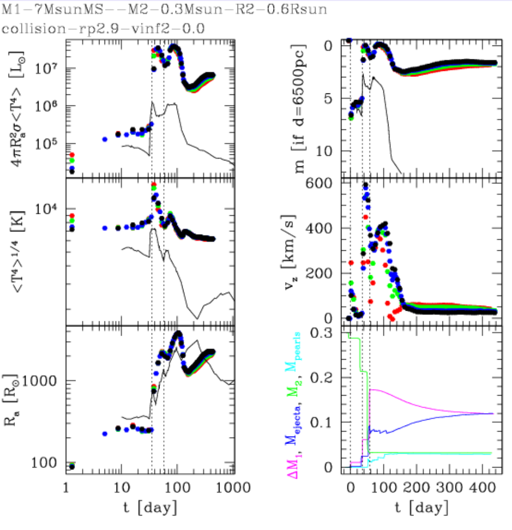
Propossed Models/Explanations:

1. Internal combustion and runaway (Atypical nova outburst, Thermal pulse of a dying star, Thermonuclear event within a massive supergiant –helium flash–, ...).
2. Accretion material (Planetary capture event).
3. Possible merger (Mergerburst,...)


May 20, 2002
September 2, 2002
October 28, 2002
December 17, 2002
Light Echo from Star V838 Monocerotis
Hubble Space Telescope • Advanced Camera for Surveys
NASA, ESA and H.E. Bond (STScI) • STScI-PRC03-10

Simulating the merger of «V838 MONO» + "companion"





- in collab w/F.Antonini, J.Faber, J.Lombardi (in prep.)

SPH Technique

- Flexible and powerful tool
- (nearly)-Incompressible flows
- Mesh-free/Lagrangian formulation
- similarities with *molecular dynamics*
- SPH algorithms *slower* than finite-difference calculations, although easier to implement

Astrophysics

- Cosmology
- Galactic Dynamics
- Stellar Interactions
- chemical composition
- radiative transfer (in progress)
- BH "physics"
- GR-sph (in progress)

Other Applications

- engineering
- disrupted free surfaces
- several fluids
- elastic fractures
- thermal and matter diffusion, chemical precipitation
- special effects & graphics industry

compute • calcul CANADA

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References

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- `StarCrash`: «A Parallel SPH Code for Calculating Stellar Interactions»
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- Smoothed Particle Hydrodynamics
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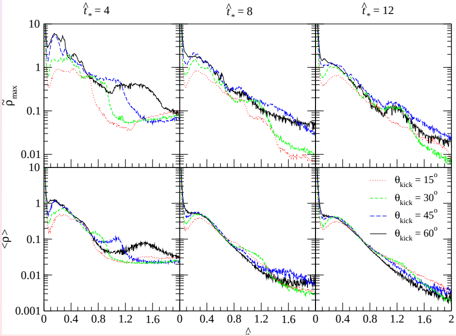
- Astrophysical SPH
  S. Rosswog
  New Astronomy Reviews, 53 (2009)
- Conservative, special-relativistic SPH
  S. Rosswog
  J.Comput.Phys, 229 (2010) 22
- SPH and Magnetohydrodynamics
  D.J. Price
  J.Comput.Phys. 231 (2012)

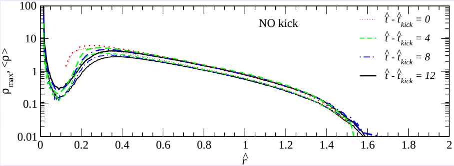
Further Resources

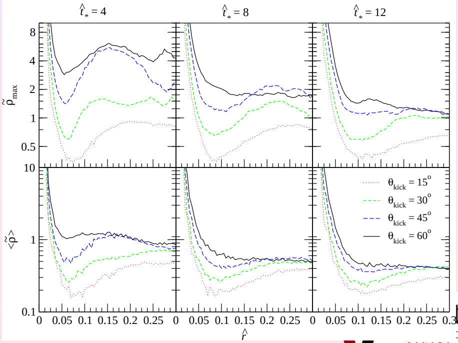
- SPH European Research Interest Community (**SPHERIC**)
  https://wiki.manchester.ac.uk/spheric
- **Gadget**: Cosmological N-body/SPH
  http://www.mpa-garching.mpg.de/gadget/
- Running your first SPH simulation
- *D.Price*'s website, including **Splash**:
  http://users.monash.edu.au/~dprice/software
- even a Python framework: **pysph**
  https://code.google.com/p/pysph/
- GPGPU: http://gpgpu.org/tag/physics-simulation

Densities

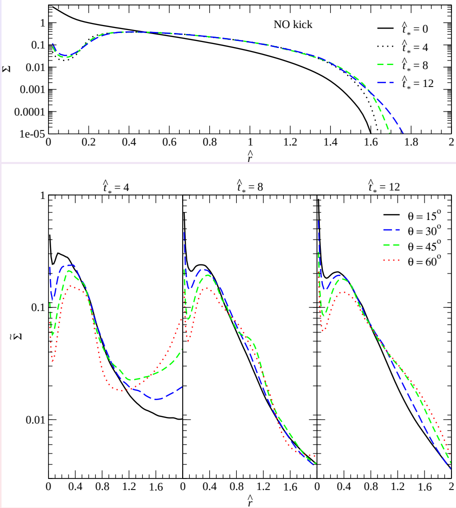
- Unkicked run does very little besides small amount of accretion
- Oblique runs show a pulse of particles heading to BH at early times
- Vertical Kicks have a persistent “gap” at the inner edge of the disk; oblique kicks fill it rapidly







Surface Density



Opacities

- no rdn.transport nor radiative cooling
- disks are hot and diffuse throughout ⇝ ≈ Thompson oppac. for ionized plasmas

$$\tau \equiv \kappa_e \Sigma \approx 3.8 \left(\frac{m_{\rm disk}}{10^4\ M_\odot}\right)\left(\frac{M_{\rm BH}}{10^8 M_\odot}\right)^{-2}\left(\frac{v_{\rm kick}}{10^8\ {\rm cm/s}}\right)^4 \tilde{\Sigma}$$

- optically thin/thick ➟ m_{disk}, M_{BH} & v_{kick}

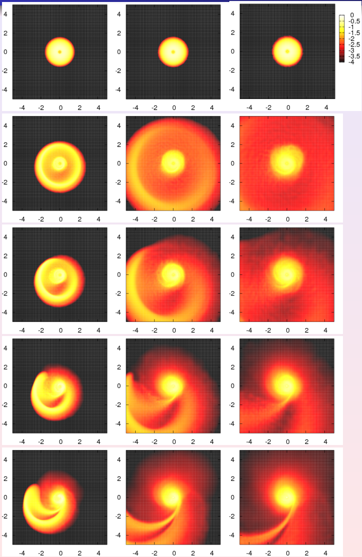
Ref. model: $m_{disk} = 10^4 M_\odot$, $M_{BH} = 10^8 M_\odot$ & $v_{kick} = 10^8$cm/s

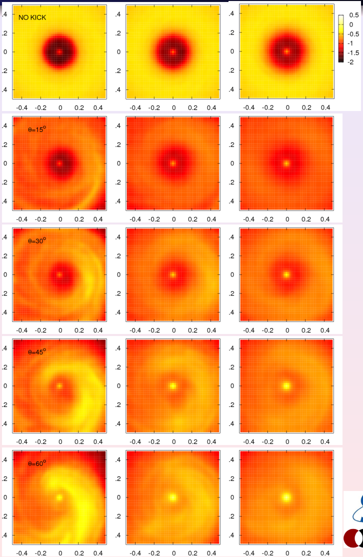
- pre-kick: $\tau > 1$ & ▸ post-kick: $\tau \lesssim 1$

Our model, predicts

- initial opt-'thin' disk ➟ opt.thin post-kick disk
- initial opt-'thick' disk ➟ slighlty less opt.thick post-kick disk

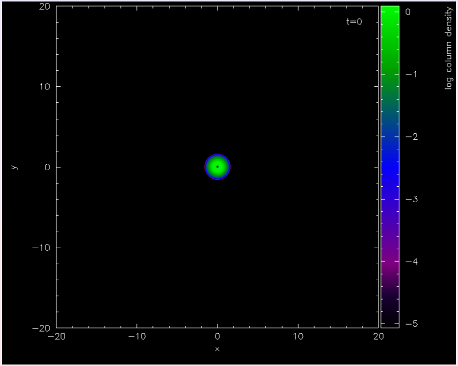
Surface Densities



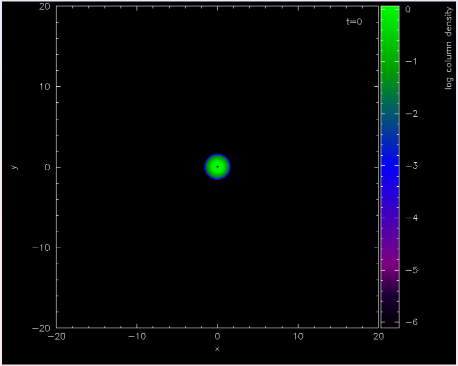


Movies available at http://lanl.arxiv.org/abs/1107.1711

$\theta = 15^\circ$

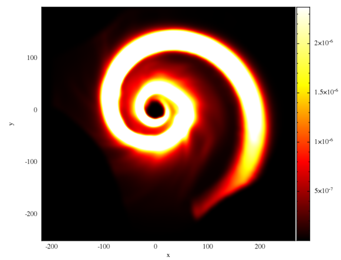


$\theta = 60^\circ$



* [MP, Faber, Lombardi – ApJ. 745 (2012) 71]

Studying TIDAL DISRUPTION EVENTS



- micro-physics (chemical composition)
- nucleosynthesis
- equation of state
- radiation transport

GR effects

- background (passive) metric
- e.g. Schwarzchild, Kerr (Paczynski-Wiita potential)
- some formulations already out there, ...
- gradual approach: “import” effects from full NR-sims, SR, passive-GR, full-GR

in collab w/F.Foucart, M.Duez, J.Faber, J.Lombardi, *and others*