Accelerate MHD

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Outline

- Magneto-hydrodynamics
- Physics problem (video)
- The algorithm of solving MHD
- Implementation on heterogeneous system
- CUDA on a mini GPU cluster
- Summary

Magneto-hydrodynamcs (MHD)

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$$ Mass conservation

$$\frac{\partial v}{\partial t} + (v \bullet \nabla) v = -\frac{1}{\rho}\nabla P - \frac{1}{4\pi\rho} B \times (\nabla \times B)$$ Momentum conservation

$$\frac{\partial}{\partial t}(\frac{1}{2}\rho v^2 + \rho\varepsilon + \frac{B^2}{8\pi}) = -\nabla(\rho v(\frac{1}{2}v^2 + \varepsilon + \frac{P}{\rho}) + \frac{B \times (v \times B)}{4\pi})$$ Energy conservation

$$\partial_t \vec{b} = \nabla \times (\vec{v} \times \vec{b})$$ Induction equation

$$\nabla \bullet \vec{b} = 0$$ No magnetic monopole

$$P = P(\rho, T)$$ Gas: equation of state

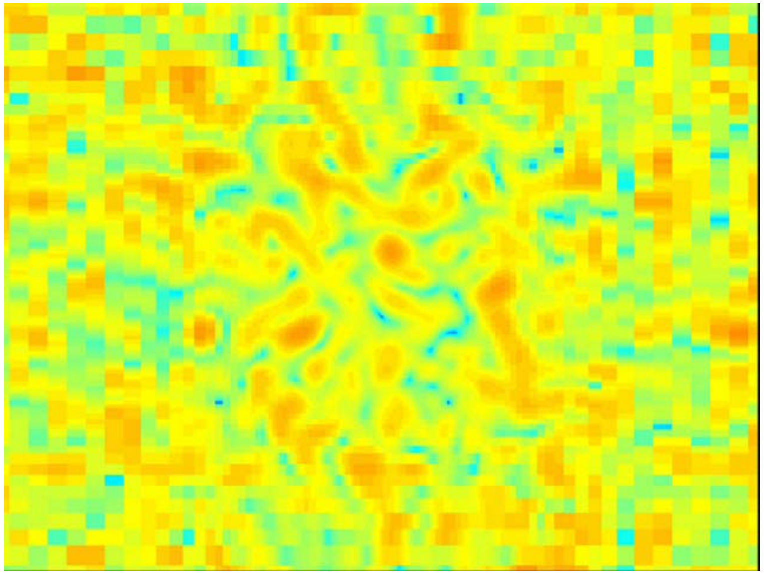
- Fluid dynamics under magnetic field

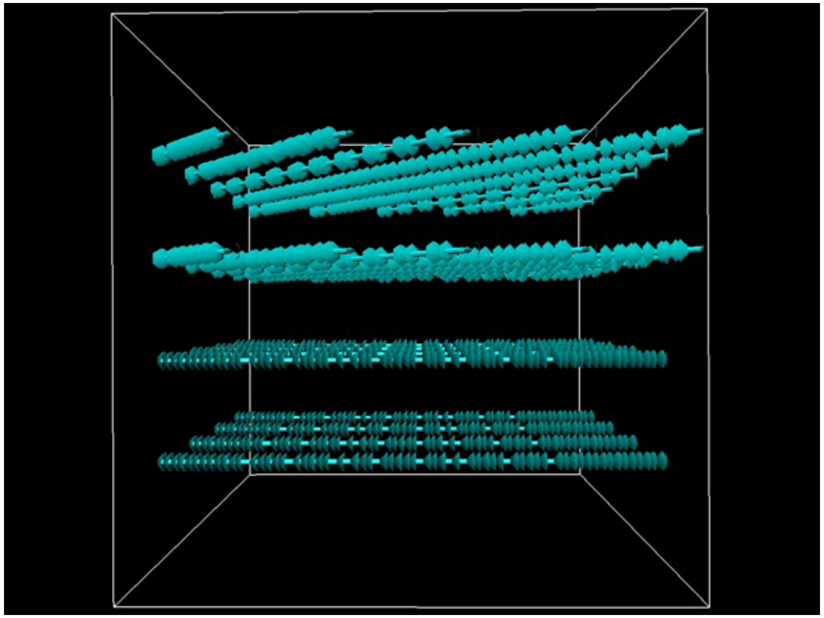
Landau & Lifshitz

MHD code

- The code was written by Ue-li Pen in 2003, and was expanded by Phil Arras, ShingKwong Wong, Hugh Merz, Matthias Liebendoerfer, Stephen Green, Bijia Pang.
- The code is a second-order accurate (in space and time) high-resolution total variation diminishing (TVD) MHD parallized code.
- Kinetic, thermal, and magnetic energy are conserved and divergent of magnetic field was kept to zero by flux constrained transport.
- The code is short and simple, easy for GPU acceleration.
- Three groups, H. Wong (arXiv:0908.4362), and H.-Y. Schive (arXiv:0907.3390), and B. Pang (arXiv:1004.1680) programmed it using CUDA on Nvidia GPU.

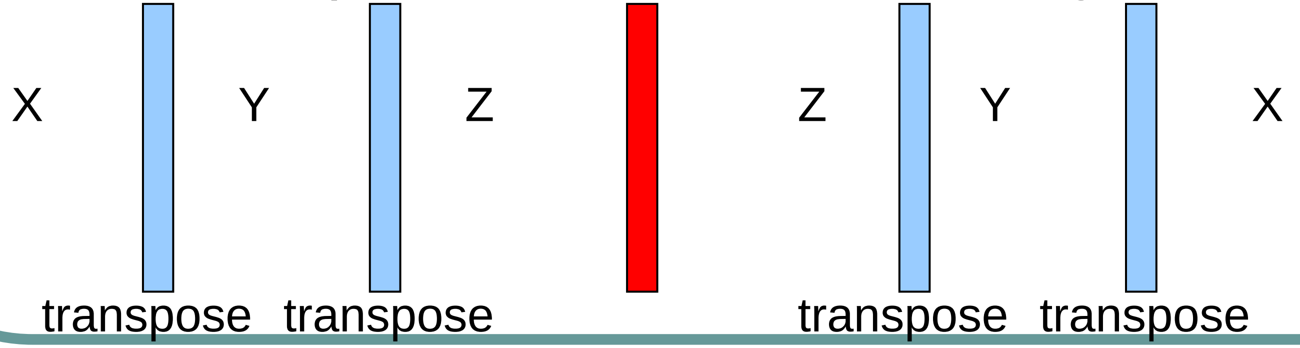
Simulation video





Algorithm of MHD code

- Finite difference + finite volume + time dependent
- u(5) fluid variable, stored on center
- b(3) magnetic variable, stored on cell face
- Second-order total variation diminishing scheme
- Dimension split for 3D box
- Fluid and magnetic update separately (1D)
- Matrix transpose is used for coalescing memory



Algorithm – fluid & magnetic

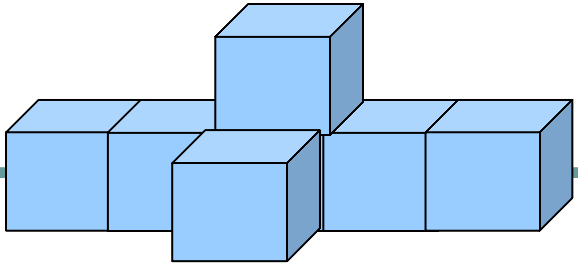
- Fluid: 1D advection equation

$$\partial_t \vec{u} + \nabla_x \vec{F} = 0$$

- Magnetic: 2D advection-constraint step. The same electro-motive force is used immediately in the constraint step to preserve zero divergent of magnetic field

$$\partial_t \vec{b} = \nabla \times (\vec{v} \times \vec{b}) \qquad \nabla \bullet \vec{b} = 0$$

Grid dependence



Pen, Arras, & Wong 2003

Heterogeneous platform

- Controlling processor + computing processors:

- CELL: Power Processor Element (PPE) + Synergistic Processing Elements (SPE)

  supported by IBM & MITACS
- GPU: CPU + unified shaders

- CELL: CELL SDK
- Nvidia: Compute Unified Device Architecture (CUDA)
- ATI: Open Computing Language (OpenCL)

The code is suited for acceleration

- The code is simple
- One dimensional update (same operation for every grid)
- Linear memory-access patterns (data transfer)

Results on different platforms

128^3 box on one CELL/GPU

Single-precision & milli-second

Architecture	x86(1)	x86(8)	Cell	N-GPU	A-GPU
Respective time	8770	1315	864	64	128
OpenCL time	N/A	6435	N/A	65	128
Peak Gflops	17	136	409.6	1030	2720
Peak GB/s	19.2	19.2	204.8	144	153.6
Power(Watts)	N/A	170	440	550	360
Code speed-up	1.0	6.7	10.2	137	68.5
Fractional speed-up	1.0	0.83	0.42	2.0	0.43
FLOPS fraction	3.1%	2.6%	1.3%	7.0%	1.3%
Bandwidth fraction	1.3%	8.8%	1.3%	24.2%	11.3%

XeonE5506@
2.13GHz

Cell blade Q22

Tesla C2050

HD 5870

Code speed-up: ratio on the platform compared to a single core x86

Fractional speed-up: ratio of code speed-up to theoretical peak performance ratio

FLOPS fraction: ratio of actual FLOPS to theoretical peak performance

Bandwidth fraction: ratio of actual data transfer to theoretical bandwidth(on-chip)

CUDA & openCL On Nvidia GPU

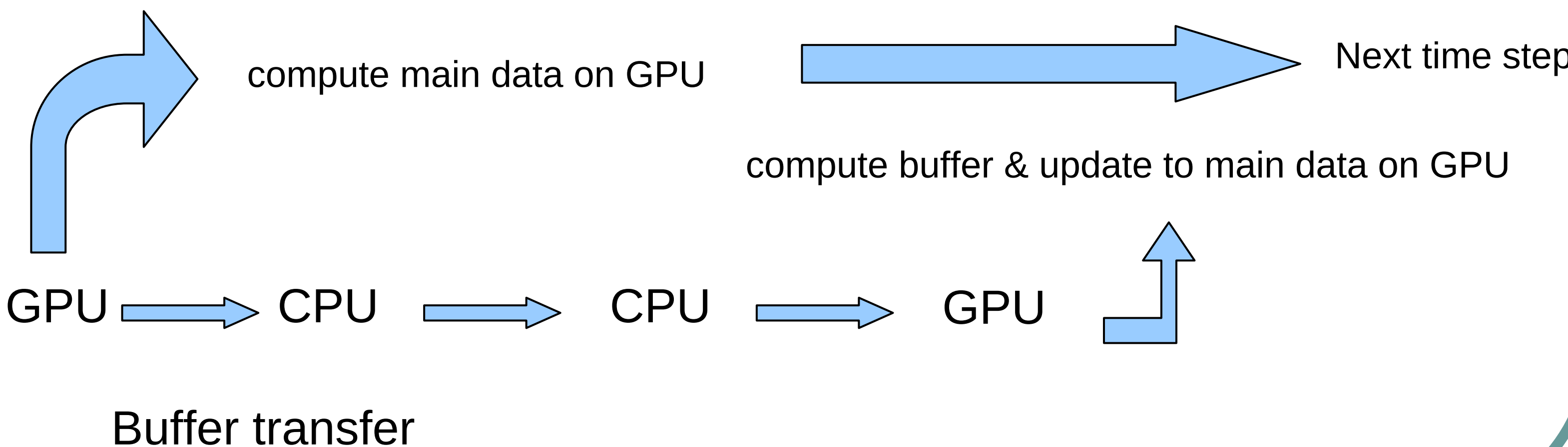
Tesla C2050

	Domain size			
Architecture	16^3	32^3	64^3	128^3
x86(1)	17.8	140	1096	8770
Nvidia (CUDA)	1.3	2.3	8.8	64
Nvidia (OpenCL)	1.5	2.5	9.3	65
Nvidia (CUDA) 2	1.9	3.7	17.9	136
Speedup (CUDA:x86)	13.1	61	125	137

Double precision

Fortran MPI + CUDA

- CUDA for Nvidia GPU
- Overcome low PCI-e bandwidth → let more data stay on GPU, less data for communication
- CPU to GPU (1 to 1)



Result on MPI + CUDA

MPI + cuda vs CPU MPI

speed-up(CUDA:openMP): 8.286/2.449=3.4

single precision (second)

time	cuda(2)	cuda(1)	fortran(2)	fortran(2) omp
two 218^3	2.449	1.489	31.08	8.286

note:
two cube for this table:
cuda(2): 1 GPU on 1 node, 2 nodes together
cuda(1): 2 GPU on 1 node
fortran(2): 1 CPU on 1 node, 2 nodes together, only fortran
fortran(2) omp: 1 CPU on 1 node, 2 nodes together, openMP fortran

lost on communication: 1.330/0.662=2

	cuda(1)	no comm
one 218^3	1.330	0.662

one cube for this table:
cuda(1): 1 GPU on 1 node, including the MPI communication
no comm: 1 GPU on 1 node, no communication

2 Tesla C1060 GPU + MPI

122^3 detail link

Summary

- Heterogeneous system can accelerate: Cell(10x), CUDA(137x), ATI(68x)
- ATI has a good theoretical peak performance, but CUDA on Nvidia perform better. (our openCL code not fully vectorized)
- CUDA & openCL perform the same on C2050
- CUDA + MPI can accelerate(3.4x to openMP)

- Future work: improve CUDA + MPI

Reference:

- H. Wong, U. Wong, X. Feng, and Z. Tang, "Magnetohydrodynamics simulations on graphics processing units," Imprint, 2009
- H.-Y. Schive, Y.-C, Tsai, and T. Chiueh, "GAMER: a GPU-Accelerated Adaptive Mesh Refinement Code for Astrophysics,"Astrophys. J. Suppl., vol. 186, pp. 457-484, 2010
- B. Pang, U. Pen, M. Perrone, "Magnetohydrodynamics on Heterogeneous architectures: a performance comparison," Imprint, 2010

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Thank you!

How to update magnetic

- Find a second-order-accurate, upwind ElectroMotiveForce V_y*B_x
- In the advection step to update b_x and then immediately use the same EMF for the constraint step to update b_y.

$$\partial_t \boldsymbol{b} = \nabla \times (\boldsymbol{v} \times \boldsymbol{b}) ,$$

$$\nabla \cdot \boldsymbol{b} = 0 ,$$

$$\partial_t b_x + \partial_y (v_y b_x) = 0$$

$$\partial_t b_y = \partial_x (v_y b_x)$$

Pen, Arras, & Wong 2003

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Upwind methods

- Upwind methods take into account the physical nature of the flow when assigning fluxes for the discrete solution.
- Excellent at capturing shocks and also highly stable.

Courant, Isaason, & Reeves (1952)

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Total variation diminishing

- Nonlinear stability condition
- Overall number of oscillations is bounded
- A strongly nonlinear flux limiter that adds just enough diffusion to prevent numerical instabilities.

$$TV(u^t) = \sum_{i=1}^{N} | u_{i+1}^t - u_i^t | \qquad TV(u^{t+\Delta t}) \le TV(u^t)$$

$$\Delta F_{n+1/2}^t = \phi(\Delta F_{n+1/2}^{L,t}, \Delta F_{n+1/2}^{R,t})$$

Harten (1983)

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Relaxing system

- Euler equation: momentum and energy fluxes depend on the pressure.
- The flow is considered as a sum of a right-moving wave u_R and a left-moving u_L.

$$u^L = \left(\frac{1 - v/c}{2}\right)u \qquad u^R = \left(\frac{1 + v/c}{2}\right)u$$

$$\frac{\partial u}{\partial t} + \frac{\partial F^R}{\partial x} - \frac{\partial F^L}{\partial x} = 0$$

Jin & Xin 1995

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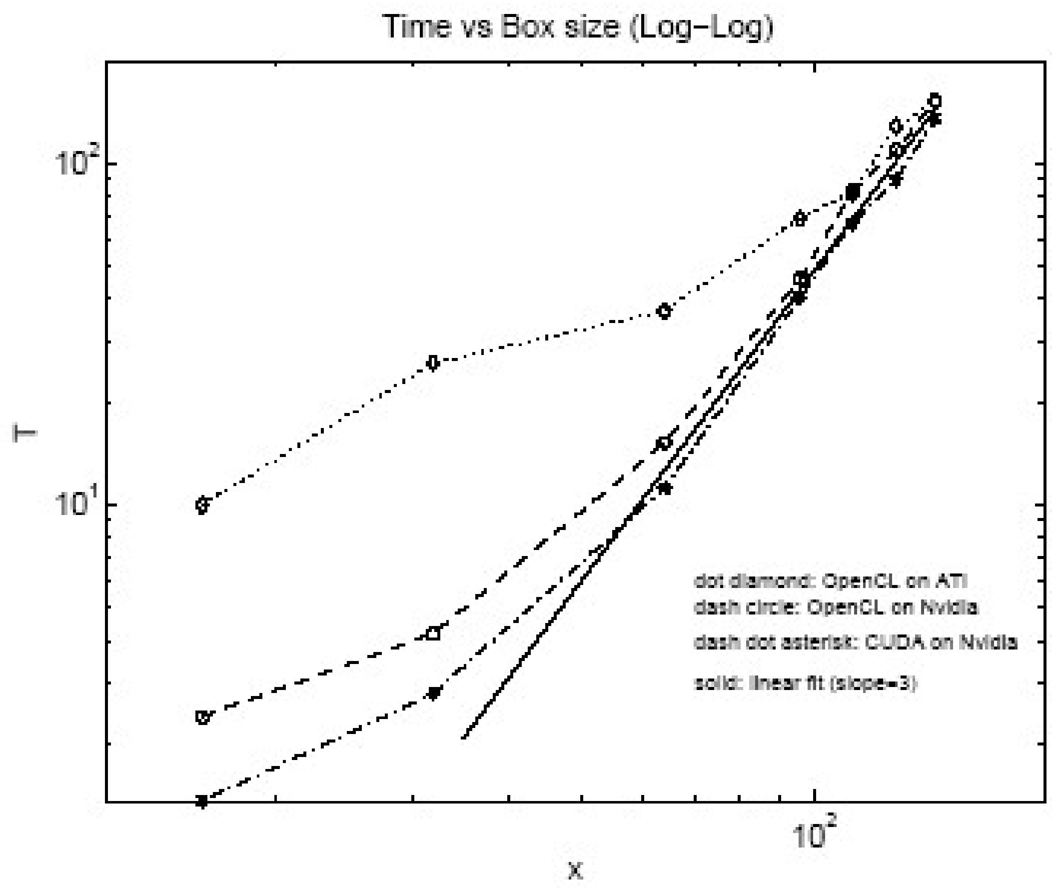
Comparison between OpenCL & CUDA



Algorithm – fluid part

- 1D advection equation

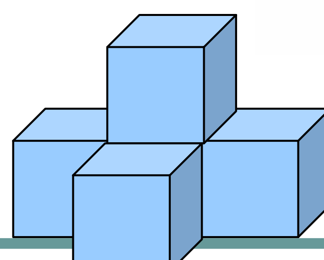
$$\partial_t \vec{u} + \nabla_x \vec{F} = 0$$

$$\boldsymbol{u} = (u_1, u_2, u_3, u_4, u_5)$$

$$(\rho, \rho v_x, \rho v_y, \rho v_z, e)$$

$$\boldsymbol{F} = \begin{pmatrix} \rho v_x \\ \rho v_x^2 + P_* - b_x^2 \\ \rho v_x v_y - b_x b_y \\ \rho v_x v_z - b_x b_z \\ (e + P_*) v_x - b_x \boldsymbol{b} \cdot \boldsymbol{v} \end{pmatrix}$$

Cell dependence



Pen, Arras, & Wong 2003

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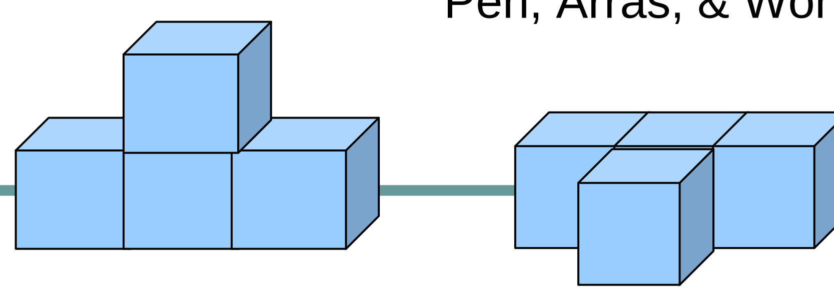
Algorithm – magnetic part

- Constrained transport(CT):
- 1. store magnetic field at cell faces;
- 2. the same electro-motive force is used immediately in the constraint step to preserve zero divergent of magnetic field.

$$\partial_t \vec{b} = \nabla \times (\vec{v} \times \vec{b}) \qquad \nabla \bullet \vec{b} = 0$$

Pen, Arras, & Wong 2003



Cell dependence

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Detail for 122^3 box

```
single precision                                    (second)

---------------------------------------------------------------
                 | cuda(1)        | no comm                      |
---------------------------------------------------------------
one 122^3        | 0.342          | 0.104                        |
---------------------------------------------------------------
one cube for this table:
cuda(1):          1 GPU on 1 node, including the MPI communication
no comm:          1 GPU on 1 node, no communication


for one 122^3, extra 230 ms for communication
including:

device to host:                     70 ms
initiate buffer communication:      40 ms
copy inside gpu:                    20 ms (overlap)
wait for communication:             10 ms
host to device:                     50 ms
buffer calculation on device:       30 ms
update main matrix data:            30 ms
```

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